# Program Overview

| Saturday June 24 | Morning Session<br>Room 108<br>Chairs：Hongjun Gao, Meihua Yang | | Afternoon Session<br>Room 108<br>Chairs: Fuke Wu, Jinqiao Duan |
|---|---|---|---|
| 8:45-9:15 | Opening Ceremony | 14:00-14:30 | Nguyen Dinh Cong |
| 9:15-9:45 | Peter Imkeller | 14:30-15:00 | Hirofumi Osada |
| 9:45-10:00 | Tea Break | 15:00-15:15 | Tea Break |
| 10:00-10:30 | Yuri Kifer | 15:15-15:45 | Hans Crauel |
| 10:30-11:00 | Peter Kloeden | 15:45-16:15 | Michael Scheutzow |
| 11:00-11:30 | Volker Wihstutz | 16:15-16:45 | Bjoern Schmalfuss |
| 11:30-12:00 | Renming Song | 16:45-17:15<br>17:15-17:45 | Ludwig Arnold |
| 12:00-14:00 | Lunch | 18:30-20:30 | Conference Dinner |

| Sunday June 25 | Morning Session<br>Room 108<br>Chairs: Bjorn Schmalfuss, Ji Li | | Afternoon Session I<br>Room 205<br>Chairs: Jian Ren, Xiaopeng Chen | Afternoon Session II<br>Room 202<br>Chairs: Jicheng Liu, Yejuan Wang |
|---|---|---|---|---|
| 8:45-9:15 | Manfred Denker | 14:00-14:30 | Jiaowan Luo | Jianhua Huang |

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|---|---|---|---|---|
| **9:15-9:45** | Yanmei Kang | **14:30-15:00** | Zhenxin Liu | Hakima Bessaih |
| **9:45-10:00** | **Tea Break** | **15:00-15:15** | **Tea Break** | |
| **10:00-10:30** | Bin Wang | **15:15-15:45** | Maria Garrido-Atienza | Jinzhi Lei |
| **10:30-11:00** | Youmin Tang | **15:45-16:15** | Gerald Trutnau | Hao Ge |
| **11:00-11:30** | Wansuo Duan | **16:15-16:45** | Gunter Ochs | Jae Kyoung Kim |
| **11:30-12:00** | Xiangsan Liang | **16:45-17:15** | Cecilia Gonzalez Tokman | Wei Wang |
| | | **17:15-17:45** | Vena Pearl Bongolan | Bin Pei |
| **12:00-14:00** | **Lunch** | **18:30-20:30** | **Dinner** | |

| **Monday June 26** | **Morning Session**<br><br>**Room 108**<br><br>**Chairs: Yiwei Zhang, Ke Yin** | | **Afternoon Session I**<br><br>**Room 205**<br><br>**Chairs: Xiaojun Wang, Xuewei Ju** | **Afternoon Session II**<br><br>**Room 202**<br><br>**Chairs: Qinbo Liu, Jintao Wang** |
|---|---|---|---|---|
| **8:45-9:15** | Anthony Roberts | **14:00-14:30** | Caishi Wang | Qigui Yang |
| **9:15-9:45** | Huaizhong Zhao | **14:30-15:00** | Daiwen Huang | Yejuan Wang |

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|---|---|---|---|---|
| **9:45-10:00** | **Tea Break** | **15:00-15:15** | **Tea Break** | |
| **10:00-10:30** | Xiaofan Li | **15:15-15:45** | Ilya Pavlyukevich | Zhongkui Sun |
| **10:30-11:00** | Jifa Jiang | **15:45-16:15** | Guanglin Rang | Liang Wang |
| **11:00-11:30** | Yongjin Wang | **16:15-16:45** | Xiaoliang Wan | Yong Chen |
| **11:30-12:00** | Xiaoyue Li | **16:45-17:15** | Guangyin Lv | Xiaopeng Chen |
| | | **17:15-17:45** | Xiaofeng Ye | Xiang Lv |
| **12:00-14:00** | **Lunch** | **18:30-20:30** | **Dinner** | |

| **Tuesday June 27** | **Morning Session**<br>**Chairs:Xiaoyue Li, Chujin Li** | **Afternoon**<br>**Chairs: Wei Zhang, Jinyue Xu, Yancai Liu, Xiaoli Chen , Pinyuan Wei** |
|---|---|---|
| **8:45-9:15** | Valery Oseledets | Panel Discussion |
| **9:15-9:45** | Peidong Liu | |
| **9:45-10:00** | **Tea Break** | |
| **10:00-10:30** | Ale Jan Homburg | |
| **10:30-11:00** | Wenxiang Sun | |

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|---|---|---|---|
| **11:00-11:30** | Wen Huang | | |
| **11:30-12:00** | Yong Xu | | |
| **12:00-14:00** | **Lunch** | **17:30-19:30** | **Dinner** |

# PROGRAM

## June 24， 2017

8:45-9:15 am: **Opening session & group photo** **Chair: Jinqiao Duan**

### Morning Session

### Chairs: Meihua Yang, Hongjun Gao

9:15 am: Peter Imkeller (Humboldt University Berlin)

On the Hausdorff dimension of a Weierstrass curve whose components are not controlled.

9:45-10:00am Tea Break

10:00 am: Yuri Kifer (Hebrew University of Jerusalem)

A Nonconventional Local Limit Theorem and Random Fourier Operators.

10:30am: Peter Kloeden (Huazhong University of Science and Technology)

Mean-square random dynamical systems and mean-square dichotomies.

11:00 am: Volker Wihstutz (University of North Carolina at Charlotte)

A Nonconventional Local Limit Theorem and Random Fourier Operators.

11:30 am: Renming Song (University of Illinois)

Potential theory of subordinate killed Levy processes.

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**Afternoon Session**

**Chairs: Fuke Wu, Jinqiao Duan**

14:00 pm: Nguyen Dinh Cong (Vietnam Academy of Science and Technology)

Generation of nonlocal fractional dynamical systems by fractional differential equations.

14:30 pm: Hirofumi Osada (Kyushu University)

15:00pm –15:15pm: Tea Break

15:15 pm: Hans Crauel (Goethe-Universitaet Frankfurt)

Minimal Random Attractors: General Results.

15:45 pm: Michael Scheutzow (Technische Universität Berlin)

Minimal Random Attractors: Examples and Counterexamples.

16:15 pm: Bjoern Schmalfuss (Friedrich-Schiller-Universität)

Local stability of stochastic differential equations driven by fbm with Hurst parameter $H\in (1/3,1/2]$

16:45 pm: Ludwig Arnold (University of Bremen)

MY CHANCES WITH CHANCE
Recollections and reflections on the occasion of my 80th birthday.

**June 25，2017**

**Morning Session**

**Chairs: Bjorn Schmalfuss, Ji Li**

8:45 am: Manfred Denker (Penn State University )

9:15 am: Yan-Mei Kang (Xi’an Jiaotong University)

Some Quantitative Analysis for Stochastic Dynamics of Gene Regulatory Networks.

9:45-10:00am Tea Break

10:00 am: Bin Wang (Tsinghua University & Chinese Academy of Sciences)

Data assimilation scheme for decadal predictions using a coupled climate system model.

10:30am: Youmin Tang ( University of Northern British Columbia)

Progress towards improving seasonal climate prediction by mathematical methods.

11:00 am: Wansuo Duan (University of Chinese Academy of Sciences)

An approach to generating mutually independent initial perturbations for ensemble forecasts: orthogonal conditional nonlinear optimal perturbations.

11:30 am: Xiangsan Liang (Nanjing Institute of Meteorology)

Causality and information flow within stochastic dynamical systems and causality analysis with time series.

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**Afternoon Session I**

**Chairs: Jian Ren, Xiaopeng Chen**

14:00 pm: Jiaowan Luo (Guangzhou University)

14:30 pm: Zhenxin Liu (Dalian Univeristy of Technology)

Poisson stable solutions for stochastic differential equations.

15:00pm –15:15pm: Tea Break

15:15 pm: Maria Garrido-Atienza (Universidad de Sevilla)

15:45 pm: Gerald Trutnau (Seoul National University)

Existence and uniqueness results for Ito-SDEs with general drifts and Sobolev diffusion coefficents.

16:15 pm: Gunter Ochs (Hochschule Darmstadt University of Applied Sciences)

Some ideas on chaotic random attractors.

16:45 pm: Cecilia Gonzalez Tokman (The University of Queensland)

A spectral approach for quenched limit theorems for random dynamical systems.

17:15 pm: Vena Pearl Bongolan (University of the Philippines Diliman)

A Stochastic Agent-based Model for Mangrove Forest Regrowth.

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**Afternoon Session II**

**Chairs: Jicheng Liu, Yejuan Wang**

14:00 pm: Jianhua Huang (National University of Defense Technology)

Dynamics of stochastic fractional Boussinesq equations.

14:30 pm: Hakima Bessaih (University of Wyoming)

On stochastic modified 3D Navier-Stokes equations with anisotropic viscosity.

15:00pm –15:15pm: Tea Break

15:15 pm: Jinzhi Lei (Tsinghua University)

15:45 pm: Hao Ge (Peking University)

Nonequilibrium thermodynamics of chemical reaction kinetics.

16:15 pm: Jae Kyoung Kim (Korea Advanced Institute of Science and Technology)

The relationship between deterministic and stochastic quasi-steady-state approximations.

16:45 pm: Wei Wang (Nanjing University)

A diffusion approximation for random nonlinear wave equation with large interaction.

17:15 pm: Bin Pei (Northwestern Polytechnical University)

Stochastic Averaging for A Class of Two-Time-Scale Systems of Stochastic Partial Differential Equations.

## June 26，2017

### Morning Session

**Chairs: Yiwei Zhang, Ke Yin**

8:45 am: Anthony Roberts (University of Adelaide )

Networks.Couple periodic-patches of a stochastic microscale to predict emergent macroscale dynamics.

9:15 am: Huaizhong Zhao (Loughborough University)

Periodic Stochastic Dynamical Systems (PeriSDS).

9:45-10:00am Tea Break

10:00 am: Xiaofan Li (Illinois Institute of Technology, USA )

Numerical simulations of macroscopic quantities for stochastic differential equations with alpha-stable processes.

10:30am: Jifa Jiang ( Shanghai Normal University)

On Limiting Behavior of Stationary Measures for Stochastic Evolution Systems with Small Noise Intensity.

11:00 am: Yongjin Wang  (Nankai University)

On a class of stochastic differential equations with local times adjustment.

11:30 am: Xiaoyue Li  (Northeast Normal University)

On Stochastic Multi-Group Lotka-Volterra Ecosystems with Regime Switching.

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**Afternoon Session I**

**Chairs: Xiaojun Wang, Xuewei Ju**

14:00 pm: Caishi Wang  (Northwest Normal University)

Stochastic Schrodinger Equations Associated with Bernoulli Annihilation and Creation Operators.

14:30 pm: Daiwen Huang

15:00pm –15:15pm: Tea Break

15:15 pm: Ilya Pavlyukevich (Friedrich Schiller University Jena)

15:45 pm: Guanglin Rang  (Wuhan University)

16:15 pm: Xiaoliang Wan  (Louisiana State University)

Efficient minimum action method for capturing the most probable transition path.

16:45 pm: Guangyin Lv  (Henan University)

17:15 pm: Xiaofeng Ye  (University of Washington)

Random Transformations and its applications to Synchronization.

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**Afternoon Session II**

**Chairs: Qinbo Liu, Jintao Wang**

14:00 pm: Qigui Yang (South China University of Technology)

14:30 pm: Yejuan Wang (Lanzhou University)

Multi-valued non-autonomous random dynamical systems.

15:00pm –15:15pm: Tea Break

15:15 pm: Zhongkui Sun (Northeastern Polytechnical University)

15:45 pm: Liang Wang (Northwestern Polytechnical University)

The response analysis of fractional-order stochastic system via generalized cell mapping method.

16:15 pm: Yong Chen (Zhejiang Sci-Tech University)

Well-posedness and large deviations for a class of SPDEs with Levy noise.

16:45 pm: Xiaopeng Chen (Shantou University)

17:15 pm: Xiang Lv (Shanghai Normal University)

Small-gain theorems for nonlinear stochastic systems with inputs and outputs II: Multiplicative white noise case.

**June 27，2017**

**Morning Session**

**Chairs: Xiaoyue Li, Chujin Li**

8:45 am: Valery Oseledets (Financial University and Moscow State University )

The Erdos measures on Euclidean space, on $\hat{Z}^n$ and Heisenberg group.

9:15 am: Peidong Liu (Peking University)

Differentiation of SRB measures for hyperbolic attractors under random perturbations.

9:45-10:00am Tea Break

10:00 am: Ale Jan Homburg (University of Amsterdam )

From random dynamics to partially hyperbolic dynamics.

10:30am: Wenxiang Sun ( Peking University)

Periodic orbits and atomic measures on non-uniformly hyperbolic systems.

11:00 am: Wen Huang (Universite Catholique de Louvain)

Measure complexity and Mobius disjointness.

11:30 am: Yong Xu (Northwestern Polytechnical University)

Random attractors for stochastic differential equations driven by two-sided Lévy processes.

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# Titles and Abstracts

## MY CHANCES WITH CHANCE

### Recollections and reflections on the occasion of my 80th birthday

**Ludwig Arnold**

**University of Bremen**

**Abstract**： This is a short personal and scientific autobiography. After a few remarks about my childhood in East Germany I describe my "Years of Apprenticeship and Travel" (Germany, USA, Canada 1957-1971). Then I discuss in more detail the "Golden Years" of Random Dynamical Systems in Bremen (1976 ff). Along the way I present some results which I believe are my essential mathematical accomplishments.

## On stochastic modified 3D Navier-Stokes equations with anisotropic viscosity

**Hakima Bessaih**

**University of Wyoming**

**Abstract**： Navier-Stokes equations in the whole space subject to an anisotropic viscosity and a random perturbation of multiplicative type is described.

By adding a term of Brinkman-Forchheimer type to the model, existence and uniqueness of global weak solutions in the PDE sense are proved.

These are strong solutions in the probability sense. The convective term given in terms of the Brinkman-Forchheirmer provides some extra regularity in the

space. As a consequence, the nonlinear term has better properties which allows to prove uniqueness. The proof of existence is performed through a control method. A Large Deviations Principle is also proven.

## A Stochastic Agent-based Model for Mangrove Forest Regrowth

**Vena Pearl Bongolan**

**University of the Philippines Diliman**

(Joint work with Andrew Vince Lorbisa, Vincent Paul Fiestadaa, Severino G. Salmo IIIb)

**Abstract**：Mangrove forests along tropical coastlines frequently suffer severe damage due to storms. The mangrove trees are extremely sensitive to environmental stressors such as water salinity and tidal inundation. There is a need for accurate modelling of such systems in order to assist in the decision-making process of environmental conservationists. The authors propose an agent-based model for the prediction of the regenerative behaviour of mangrove stands consisting of the native species and the planted or non-native species in a fragmented habitat, with the use of spatio-temporal coloured noise to simulate stochastic seedling dispersal. It uses Salmo and Juanico's model for individual mangrove growth. This study aims to model accurately the growth of mangrove populations while experiencing said factors and subject to storm damage. Recovery of a population is determined in terms of the current total forest cover compared to the total forest cover before the storm occurred.

Stochastic experiments were carried out in a virtual analogue of Bangrin Marine Protected Area in Bani, Pangasinan with a population of native mangroves (Avicennia and Sonneratia) and a larger population of planted, non-native mangroves (Rhizophora). Out of 1280 runs of various configurations, the native genera always fully recovered within 5.65 to 6.84 years. The planted genera only fully recovered 6 times out of 1280. Within 4.53 to 5.45 years, the planted mangroves were only able to regain around 66.16% to 71.30% of their total pre-storm cover.

**Minimal Random Attractors: General Results**

**Hans Crauel**

**Goethe-Universitaet Frankfurt**

(Joint work with Michael Scheutzow )

**Abstract**: Given a random dynamical system and a family of subsets of its state space, an attractor for this family is a compact, strictly invariant set which attracts every element of the family. What attraction is concerned one may distinguish between pullback attraction and forward attraction, both of which imply weak attraction. Common examples are the family of all compact sets or even the family of all bounded sets (yielding set attractors) and all finite sets (yielding point attractors). While a set attractor is unique almost surely, general families of sets to be attracted may fail to have a unique attractor. Always assuming existence of a corresponding attractor, the present work establishes existence of a unique minimal attractor for general families of (possibly non-deterministic) sets both for pullback attraction and for weak attraction. A corresponding result for forward attraction fails to hold true. Michael Scheutzow will in his lecture give an example for the existence of forward attractors none of which is minimal. Also he will provide several examples of minimal pullback and weak attractors.

**Well-posedness and large deviations for a class of SPDEs with Levy noise**

**Yong Chen**

**Zhejiang Sci-Tech University**

**Abstract**： In this paper, a class of stochastic partial differential equations (SPDEs) with L\'evy noise is concerned. Firstly, the local well-posedness is established by the iterative approximation. Then the large deviation principle (LDP) for the regularized SPDEs is obtained by the weak convergence approach. To get the LDP for SPDEs here, an exponential equivalence of the probability measures is proved. The results can be applied to some types of SPDEs such as stochastic Burgers equation, stochastic b-family equation,

stochastic modified Novikov equation and stochastic $\mu$-Hunter-Saxton equation.

### Generation of nonlocal fractional dynamical systems by fractional differential equations

**Nguyen Dinh Cong**

**Vietnam Academy of Science and Technology**

(Joint work with H.T. Tuan)

**Abstract**: We show that any two trajectories of solutions of a one-dimensional fractional differential equation (FDE) either coincide or do not intersect each other. In contrary, in the higher dimensional case, two different trajectories can meet. Furthermore, one-dimensional FDEs and triangular systems of FDEs generate nonlocal fractional dynamical systems, whereas a higher dimensional FDE does, in general, not generate a nonlocal dynamical system.

### An approach to generating mutually independent initial perturbations for ensemble forecasts: orthogonal conditional nonlinear optimal perturbations

**Wansuo Duan**

**University of Chinese Academy of Sciences**

(Joint work with Zhenhua Huo)H.T. Tuan

**Abstract**: Conditional nonlinear optimal perturbation (CNOP) is the initial perturbation that satisfies a certain physical constraint and causes the largest nonlinear evolution at prediction time. To yield mutually independent initial perturbations in ensemble forecasts, orthogonal CNOPs are developed. Orthogonal CNOPs are then applied to a Lorenz-96 model to generate initial perturbations for ensemble forecasting, as compared with orthogonal singular vectors (SVs). When the initial analysis errors are fast-growing, the ensemble forecasts generated by orthogonal CNOPs of the control forecasts perform

much more skillfully. Nevertheless, for slow-growing initial analysis errors, the ensemble forecasts generated by orthogonal SVs achieve higher skill when the ensemble initial perturbations are large, whereas the ensemble forecasts generated by orthogonal CNOPs achieve almost the same forecast skill as those generated by orthogonal SVs when the ensemble initial perturbations are sufficiently small. The initial analysis errors that possess much faster growth behavior are easily influenced by nonlinearity, and extreme events (“extreme” here refers to “strong”), due to strong nonlinear instability, may be much more likely to cause fast growth of initial analysis errors. Therefore, the ensemble forecasts generated by orthogonal CNOPs may have higher skill than those generated by orthogonal SVs for extreme events; in particular, the ensemble forecasts generated by orthogonal CNOPs, compared with those generated by orthogonal SVs, require a much small number of ensemble members to achieve high skill. Therefore, orthogonal CNOPs may provide another useful technique to generate initial perturbations for ensemble forecasting.

**From random dynamics to partially hyperbolic dynamics**

**Ale Jan Homburg**

**University of Amsterdam**

**Abstract**: I'll discuss iterated function systems of diffeomorphisms on compact manifolds with a focus on synchronization by noise and on-off intermittency. The theory will be connected to the seemingly different context of Fubini's nightmare in stably ergodic diffeomorphisms.

**Nonequilibrium thermodynamics of chemical reaction kinetics**

**Hao Ge**

**Peking University**

**Abstrac**：Nonequilibrium thermodynamics and statistical physics in terms of stochastic models entered a stage of vigorous development since 1970s, which well fit the development of advanced experimental techniques in modern physical chemistry and biochemistry. I will discuss our recent stochastic approaches to investigate the nonequilibrium thermodynamics of chemical reaction systems. We show that the entropy production rate can be decomposed into the housekeeping heat and the decreasing rate of relative entropy, both of which are nonnegative, followed by a more stronger version of Clausius inequality. We further proved that in the macroscopic limit by merely allowing the molecular numbers to infinite, a generalized macroscopic free energy and its balance equation emerge. The balance equation is valid generally in isothermal driven systems. A general fluctuation dissipation theorem for stochastic reaction kinetics is also proved. Such an emergent ``law'' is independent of underlying kinetic details. The mathematical theory illustrates how a novel macroscopic dynamic law can emerge from the mesoscopic kinetics in a multi-scale system.

## Dynamics of stochastic fractional Boussinesq equations

**Jianhua Huang**

**National University of Defense Technology**

**Abstract**： In this talk, we present some results on stochastic fractional Boussinesq equations (SFBEs). The existence of random attractor for SFBEs driven by Gaussian noise are established under some suitable assumptions. The global well-posedness of SFBEs driven by Levy noise are obtained. Finally, the existence of mild solution for Caputo-type time fractional stochastic nonlocal Boussinesq equation are also established.

## Measure complexity and Mobius disjointness

**Wen Huang**

**Universite Catholique de Louvain**

**Abstract**: We will review some progress about Sarnak's Mobius disjointness conjecture by the measure complexity. Some examples incuding (Quasi)-discrete spectrum systems and skew product maps on torus over a rotation of the circle will be discussed.

## On the Hausdorff dimension of a Weierstrass curve whose components are not controlled

**Peter Imkeller**

**Humboldt University Berlin**

(Joint work with G. dos Reis (U Edinburgh) and O. Pamen (U Liverpool and AIMS Ghana))

**Abstract**: We investigate geometric properties of Weierstrass curves with two components, representing lacunary series based on trigonometric functions. They are seen to be $\frac{1}{2}-$ Holder continuous, do not possess a L´evy area, and are not (para-)controlled with respect to each other in the sense of our recent Fourier analytic approach of rough path analysis. Their graph is represented as an attractor of a smooth random dynamical system in 4-dimensional Euclidean space. Our argument that the curve is space filling (has Hausdorff dimension 2) is in the spirit of Keller's adaptation of Ledrappier-Young's approach of the Hausdorff dimension of attractors.

## On Limiting Behavior of Stationary Measures for Stochastic Evolution Systems with Small Noise Intensity

**Jifa Jiang**

**Shanghai Normal University**

（Joint work with Dr. Chen Lifeng and Profs. Dong Zhao and Zhai Jianliang)

**Abstract**: The limiting behavior of stochastic evolution processes with small noise intensity ϵ is investigated in distribution-based approach. Let $\mu^\epsilon$ be stationary measure for stochastic process $X^\epsilon$ with small ϵ and $X^0$ be a semiflow on a Polish space. Assume that $\{\mu^\epsilon : 0 < \epsilon \le \epsilon^0\}$ is tight. Then all their limits in weak sense are X0－invariant and their supports are contained in Birkhoff center of $X^0$. Applications are made to various stochastic evolution systems, including stochastic ordinary differential equations, stochastic partial differential equations, stochastic functional differential equations driven by Brownian motion or Lévy process. Examples for SODEs are found to show that their limiting measures are supported at saddles.

## A Nonconventional Local Limit Theorem and Random Fourier Operators

**Yuri Kifer**

**Hebrew University of Jerusalem**

Abstract. We exhibit a local limit theorem for nonconventional sums $S_N = \sum_{n=1}^{N} F(\xi_n, \xi_{2n}, ..., \xi_{\ell n})$ where $\{\xi_n\}$ is a Markov chain or it is generated by certain dynamical systems such as mixing subshifts of finite type. The proof leads naturally to the study of iterates of random Fourier operators and to a version of the Ruelle-Perron-Frobenius theorem for them.

## Mean-square random dynamical systems and mean-square dichotomies

**Peter Kloeden**

Huazhong University of Science and Technology

Mean-square random dynamical systems are formulated as nonautonomous dynamical systems on the time-variable state spaces $\in L_2[\Omega, \mathcal{F}_t; \mathbb{R}^d]$, $t \in \mathbb{R}$, of nonanticipative mean-square random variables. They are generated by meanfield stochastic differential equations. Many concepts from nonautonomous dynamical systems carry over, e.g., a mean-square attractor is a pulback attractor. However, lack of compactness criteria for the spaces $L_2[\Omega, \mathcal{F}_t; \mathbb{R}^d]$ make it difficult to apply the theory of nonautonomous dynamical systems. Mean-square dichotomies are defined for linear meanfield SDE and the corresponding spectrum found and is used to illustrate the bifurcation of a zero solution to a nontrivial mean-square attractor.

## The relationship between deterministic and stochastic quasi-steady-state approximations.

Jae Kyoung Kim

Korea Advanced Institute of Science and Technology

**Abstract**：The quasi steady-state approximation (QSSA) is frequently used to reduce deterministic models of biochemical networks. The resulting equations provide a simplified description of the network in terms of non-elementary reaction functions (e.g. Hill functions). Such deterministic reductions are frequently a basis for heuristic stochastic models in which non-elementary reaction functions are used as propensities of Gillespie algorithm. Despite the popularity of this heuristic stochastic simualtions, it remains unclear when such stochastic reductions are valid. In this talk, I will present conditions under which the stochastic models with the non-elementary propensity functions accurately approximate the full stochastic models. If the validity condition is satisfied, we can perform accurate and computationally inexpensive stochastic simulation without converting the non-elementary functions to the elementary functions (e.g. mass action kinetics).

## Some Quantitative Analysis for Stochastic Dynamics of Gene Regulatory

**Networks**

**Yan-Mei Kang**

**Xi’ an Jiaotong University**

**Abstract**: Since the double-helix structure of DNA was discovered in the middle of last century, genetic regulation networks have been designed as digital circuits or devices to emulate the protein-DNA interactions in fundamental cellular processes. In the recent few decades, more and more experimental progresses have been implemented to make the gene regulation networks amenable to quantitative analysis. Here I will talk about some of our work towards this regard.

Genetic regulation networks can be described as differential dynamical systems of stochasticity, non-negativeness and rationality. Note that the state variables in gene regulatory networks represent the concentration of reactants, which cannot physically be negative, so the gene regulatory networks are nonnegative system in general. Due to the Michaelis – Menten or Hill form reaction rate, the genetic regulatory systems are usually governed by differential dynamical systems of rational vector field. The stochasticity of the gene regulatory networks is determined by the essence of biochemical reactions. The process of Gene regulation including transcription and translation consists of a series of biochemical reactions. Different from the ordinary chemical reactions, the number of the involving reactants or DNA molecules is low and random in the biochemical reactions, thus this sparsity and randomness makes the fluctuations non-negligible and intrinsic in gene regulatory networks. Actually, the significance of randomness has been observed in a plenty of clinical and experimental researches about expression switches, cellular differentiation, phenotypical variety and genetic disease.

Taking the stochasticity and the rationality into account, we take a lambda expression autoregulation model driven by multiplicative and additive noises as example to extend the Gaussian moment method from nonlinear stochastic systems of polynomial vector field to noisy biochemical systems of rational polynomial vector field. Using the extended method, we disclose the phenomenon of stochastic resonance. It is found that the transcription rate can inhibit the stochastic resonant effect, but the degradation rate may take an inverse effect.

Taking the stochasticity and the non-negativeness into account, we model the abnormal fluctuation in gene regulation with the non-Gaussian noise governed by a nonlinear Ornstein-Uhlenbeck process. By imposing an absorbing boundary condition, we analytically deduce the mean first passage times of a gene transcriptional regulatory system driven by additive non-Gaussian noise

based on path integral and singular perturbation analysis. We further explore the effect of non-Gaussian noise on stochastic resonance by a novel combination of adiabatic elimination and linear response approximation.

### Poisson stable solutions for stochastic differential equations

**Zhenxin Liu**

**Dalian Univeristy of Technology**

**Abstract**: In this talk, we will discuss the problem of Poisson stability (in particular stationarity, periodicity, quasi-periodicity, Bohr almost periodicity, Bohr almost automorphy, Birkhoff recurrence, almost recurrence in the sense of Bebutov, Levitan almost periodicity, pseudo-periodicity, pseudo-recurrence, Poisson stability) of solutions for semi-linear stochastic equations by the comparability method.

### Causality and information flow within stochastic dynamical systems and causality analysis with time series

**Xiangsan Liang**

**Nanjing Institute of Meteorology**

**Abstract**：Causality analysis is a fundamental problem of interest in a wide variety of disciplines (even in philosophy). In particular, it has been identified as ‘one of the biggest challenges’ in the science of big data. Now, given two time series, can one tell, in a rigorous and quantitative sense, the cause and effect between them? In this talk, we will show that the answer is positive, and, within the framework of stochastic dynamical systems, the formalism can be rigorously established ab *initio*, rather than axiomatically proposed as an ansatz. The principle of nil causality that reads, an event is not causal to another if the evolution of the latter is independent of the former, which the classical empirical/half-empirical transfer entropy analysis and Granger causality test fail to verify in many situations, turns out to be a proven theorem here. Formulas in closed form have been obtained for systems such as the baker transformation, Hénon map, stochastic potential flow, to name but a few.

For linear stochastic systems, the resulting formula is concise in form, and its maximum likelihood estimator involves only the common statistics namely sample covariance; a corollary is that causation implies correlation, but not vice versa, resolving the long-standing philosophical debate over causation versus correlation. A news report in *PhysicsToday* is referred to the link below.

The above formula has been validated with touchstone series purportedly generated with one-way causality that defies the classical approaches such as Granger causality test and transfer entropy analysis. It has also been applied successfully to the investigation of a variety of real problems. Through a simple analysis with the stock series of IBM and GE, an unusually strong one-way causality is identified from the former to the latter in their early era, revealing to us an old story, which has almost faded into oblivion, about "Seven Dwarfs" competing with a "Giant" for the computer market.

Another example presented here regards the cause-effect relation between the two climate modes, El Niño and Indian Ocean Dipole (IOD). In the third example, an unambiguous one-way causality is found between CO2 and the global mean temperature anomaly. While it is confirmed that CO2 indeed drives the recent global warming, on paleoclimate scales the cause-effect relation may be completely reversed. Also will be presented are the identification of a simple pattern underlying a chaotic attractor, and a preliminary study of the causal structure in the near-wall turbulence.

## On Stochastic Multi-Group Lotka-Volterra Ecosystems with Regime Switching

**Xiaoyue Li**

**Northeast Normal University**

**Abstract**: Focusing on stochastic dynamics involving continuous states as well as discrete events, this paper investigates dynamical behaviors of stochastic multi-group Lotka-Volterra model with regime switching. The contributions of the paper lie on: (a) giving the sufficient conditions of stochastic permanence for generic stochastic multi-group Lotka-Volterra model, which are much weaker than the existing results in the literature; (b) obtaining the stochastic strong permanence and ergodic property for the mutualistic systems; (c) establishing the almost surely asymptotic estimate of solutions. These can explain some recurring phenomena in practice and reveal the fact that regime switching can suppress the impermanence. A couple of examples and

numerical simulations are given to illustrate our results.

### Numerical simulations of macroscopic quantities for stochastic differential equations with alpha-stable processes

**Xiaofan Li**

**Illinois Institute of Technology, USA**

**Abstract：** The mean first exit time, escape probability and transitional probability density are utilized to quantify dynamical behaviors of stochastic differential equations with non-Gaussian, $\alpha$-stable type Lévy motions. Taking advantage of the Toeplitz matrix structure of the time-space discretization, a fast and accurate numerical algorithm is proposed to simulate the nonlocal Fokker-Planck equations on either a bounded or infinite domain. Under a specified condition, the scheme is shown to satisfy a discrete maximum principle and to be convergent. The numerical results for two prototypical stochastic systems, the Ornstein-Uhlenbeck system and the double-well system are shown.

### Differentiation of SRB measures for hyperbolic attractors under random perturbations

**Peidong Liu**

**Peking University**

(Joint work with Kening Lu and Zeng Lian)

**Abstract：** Linear response formulas arise from needs in statistical mechanics. We will give an introduction to a general but formal linear response formula (mainly due to Ruelle) in dynamical systems, and then discuss the cases of hyperbolic attractors and their random perturbations.

## Small-gain theorems for nonlinear stochastic systems with inputs and outputs II: Multiplicative white noise case.

**Xiang Lv**

**Shanghai Normal University**

(Joint work with Prof. Jifa Jiang)

**Abstract**: This paper is a continuation of the paper [SIAM J. Control Optim., 54 (2016), pp. 2383–2402], which focuses on exploring the global stability of nonlinear stochastic feedback systems on the nonnegative orthant driven by multiplicative white noise and presenting a couple of small-gain results. We investigate the dynamical behavior of pull-back trajectories for stochastic control systems and prove that there exists a unique globally attracting positive random equilibrium for those systems whose output functions either possess bounded derivatives or are uniformly bounded away from zero. Our results can be applied to well-known stochastic Goodwin negative feedback system, Othmer-Tyson positive feedback system and Griffith positive feedback system as well as other stochastic cooperative, competitive and predator-prey systems.

## The Erdos measures on Euclidean space, on $\hat{Z}^n$ and Heisenberg group

**Valery Oseledets**

**Financial University and Moscow State University**

We study the distribution of the random variable :

$$\zeta = A^{-1}\xi_1 + A^{-2}\xi_2 + ...,$$

where $\xi_k \in Z^n$ are independent identically distributed random variables, $0 < P(\xi_1 = 0) < 1$, the expanding matrix $A \in M_n(Z)$.

We will call the distribution of the random variable $\zeta$ the *Erdős measure* on the space $R^n$.We will call the distribution of the random variable $\zeta$ the *Erdős measure* on the space $R^n$. This measure is some self-similar measure.

Another measure is the distribution of the random variable:

$$\widehat{\zeta} = \xi_0 + A\xi_1 + A^2\xi_2 + ...,$$

Here $\widehat{\zeta} \in \widehat{Z^n}$, where $\widehat{Z}^n$ is the profinite completion of $Z^n$ with respect to $Z^n > AZ^n > A^2Z^n > .....$

We will call the distribution of the random variable $\zeta$ the *Erdős measure* on the group $\widehat{Z^n}$.

For Heisenberg group we consider the distribution of the random variable

$$\zeta_H = \prod_{k=1}^{\infty} \Phi^{-k}\xi_k,$$

where $\xi_k \in L$ are independent identically distributed random variables, $0 < P(\xi_1 = Id) < 1$, L is discrete Heisenberg group, $\Phi$ is expanding automorpism of Heisenberg group and $\Phi(L) < L$.

We will call the distribution of the random variable $\zeta_H$ the *Erdős measure* on Heisenbeg group.

We obtain a necessary and sufficient condition for the absolute continuity of these distributions.

We use the notions of invariant *Erdős* measure on the torus, the invariant *Erdős measure* on the compact abelian group $\widehat{Z^n}$ and the invariant *Erdős measure* on Heisenberg nilmanifold.

We show a connection of these invariant measures with the functions of countable stationary Markov chains. In the case when $|\{j : p_j \neq 0\}| < \infty$ we establish the relation of these invariant measures to the case of finite stationary Markov chains. .

## Some ideas on chaotic random attractors.

**Gunter Ochs**

**Hochschule Darmstadt University of Applied Sciences**

**Abstract：** We consider the situation of a random dynamical system with a global attractor which contains a deterministic fixed point. If this fixed point has a positive and a negative Lyapunov exponent, previous case studies of the stochastic Duffing-van der Pol oscillator by Keller and Ochs have indicated some sort of chaotoc dynamics possibly due to random homoclinic points.

In this talk we present an approach to find general conditions, under which it is possible to prove transversal intersections between stable and unstable manifolds of the fixed point, which leads to chaotic dynamics. In addition to our theoretical considerations we will show numerical examples.

**Stochastic Averaging for A Class of Two-Time-Scale Systems of Stochastic Partial Differential Equations.**

**Bin Pei**

**Northwestern Polytechnical University**

(Joint work with Yong Xu, George Yin)

**Abstract**：We focus on systems of stochastic partial differential equations that have a slow component driven by a fractional Brownian motion and a fast component driven by a fast-varying diffusion. We establish an averaging principle in which the fast-varying diffusion process acts as a ``noise'' and is averaged out in the limit. The slow process is shown to have a limit in the mean sense, which is characterized by the solution of a stochastic partial differential equation driven by a fractional Brownian motion whose coefficients are averages of that of the original slow process with respect to the stationary measure of the fast-varying diffusion. This averaging principle paves a way for reduction of computational complexity. The implication is that one can ignore the complex original systems and concentrates on the average systems instead.

**Couple periodic-patches of a stochastic microscale to predict emergent macroscale dynamics**

**Anthony Roberts**

**University of Adelaide**

**Abstract**：Molecular simulations, while in principle deterministic on the microscale, are chaotic and so effectively stochastic on meso/macro-scales.Many other microscale simulations are directly stochastic.Furthermore, as for many microscale systems, molecular simulations are much the easiest to code with periodic boundary conditions in space. We seek to distribute many small patches of such stochastic microscale periodic simulators over a macroscale grid. Then the research aim is find a coupling between the small periodic-patches in order to effectively and cheaply simulate the macroscale dynamics. A stochastic slow manifold analysis of the system of stochastic periodic-patches indicates that a proportional control applied within action regions of each patch can be effective.The analysis suggests Lagrange interpolation of core-patch averages provides a useful control. A pilot implementation of the coupling to a small triply-periodic patch of atomistic simulation indicates that the scheme usefully predicts the emergent heat diffusion.

## Local stability of stochastic differential equations driven by fbm with Hurst parameter $H\in (1/3,1/2]$

**Bjoern Schmalfuss**

**Friedrich-Schiller-Universität**

**Abstract：** We use the method of compensated derivatives to formulate and to solve an SDE with Hurst parameter $H\in (1/3,1/2]$. In addition we formulate conditions on the coefficients such that the solution of this equation is locally exponential stable.

## Minimal Random Attractors: Examples and Counterexamples

**Michael Scheutzow**

**Technische Universität Berlin**

（Joint work with Hans Crauel ）

**Abstract：** We complement the results on the existence of minimal random attractors in the talk by Hans Crauel with several examples. We provide an example showing that a minimal forward attractor may not exist in spite of the fact that (many) forward attractors exist. We also show that a minimal pullback point attractor may be significantly different from the closure of the union of all Omega-limit sets of all singletons.

**Potential theory of subordinate killed Levy processes**

**Renming Song**

**University of Illinois**

**Abstract**: Let $Z$ be a subordinate Brownian motion in $R^d$, $d \geq 3$, via a subordinator with Laplace exponent $\varphi$. We kill the process $Z$ upon exiting a bounded open set $D \subset R^d$ to obtain the killed process $Z^D$, and then we subordinate the process $Z^D$ by a subordinator with Laplace exponent $\psi$. The resulting process is denoted by $Y^D$. Both $\varphi$ and $\psi$ are assumed to satisfy certain weak scaling conditions at infinity.

We study the potential theory of $Y^D$, in particular its boundary theory. If $D$ is a smooth open set, we establish sharp two-sided estimates on the Green function of $Y^D$ and prove the boundary Harnack principle with explicit decay rate near the boundary of $D$ .These results show that the boundary behavior of $Y^D$ is completely determined by $\varphi$. On the other hand, in the interior of $D$, the behavior of $Y^D$ is determined by the composition $\psi \circ \varphi$. Under an additional condition on $\psi$, we also establish sharp two-sided estimates on the jumping kernel of $Y^D$.

In case that $D$ is a -fat bounded open set, we show that the Harnack inequality holds, and prove a boundary Harnack principle for non-negative functions harmonic in a smooth open set E strictly contained in $D$ . If, in

addition, $D$ satis es the local exterior volume condition, then we show that the Carleson estimate holds.

**Periodic orbits and atomic measures on non-uniformly hyperbolic systems**

**Wenxiang Sun**

**Peking University**

**Abstract**: A hyperbolic measure and its Lyapunov exponents can be approximated by atomic measures and their Lyapunov exponents rspectively.

**Progress towards improving seasonal climate prediction by mathematical methods.**

**Youmin Tang**

**University of Northern British Columbia**

**Abstract**: In this talk, we will present some progresses in improving seasonal climate predictions by using more advanced mathematical methods. The first example is to rely on the basic properties of stochastic theory to develop an efficient technique for the extraction of climatically relevant singular vectors (CSV) in the presence of weather noise. Emphasis is placed on the applications of the CSV in seasonal climate predictions and to construct optimal ensemble climate predictions. The results indicates that the CSVs can well characterize the optimal error growth of the climate predictions and lead to better ensemble predictions than traditional time lag (TLE) method. The second example is about our recent progress in the state estimate of state-space models with applications of Bayesian-based algorithms. A

simplified algorithm of Sigma-point Kalman filter is develop to deal with the state estimation of high-dimensional systems like atmospheric and oceanic general circulation models.

## Existence and uniqueness results for Ito-SDEs with general drifts and Sobolev diffusion coefficents

**Gerald Trutnau**

**Seoul National University**

(Joint work with Haesung Lee (Seoul National University) and Minjung Gim (NIMS, South Korea)).

**Abstract：** Using the Dirichlet form method, we construct weak solutions to SDEs in Eu-clidean space with general measurable coe cients up to their explosion times. Restricting the assumptions on the coe cients to the ones of [Zhang, X., Strong solutions of SDES with singular drift and Sobolev di usion coe cients, SPA 2005] or [Krylov, N. V., R • ockner, M., Strong solutions of stochastic equations with singular time dependent drift, PTRF 2005], we obtain by the uniqueness results of the mentioned papers that our weak solutions coin-cide with the pathwise unique and strong solutions constructed there up to their explosion times. The use of our approach is that we can now additionally use analytic non-explosion criteria to obtain pathwise unique and strong solutions up to in nity. In particular, analytic non-explosion and recurrence criteria will be presented.

## A spectral approach for quenched limit theorems for random dynamical systems

**Cecilia Gonzalez Tokman**

**The University of Queensland**

(Joint work with D. Dragicevic, G. Froyland and S. Vaienti)

**Abstrac:** Random or non-autonomous dynamical systems provide very flexible models for the study of forced or time-dependent systems, with driving mechanisms allowed to range from deterministic forcing to stationary noise. In this talk, we present a spectral approach to the study of non-autonomous dynamics, developed in the last decade via multiplicative ergodic theory. We then show how spectral methods can be used to establish quenched limit theorems for a class of non-autonomous dynamical systems.

## Noise induced rotation versus resonance in randomly perturbed linear systems.

**Volker Wihstutz**

**University of North Carolina at Charlotte**

**Abstract:** Surveying older and newer results on physically realizable linear systems with multiplicative noise, we consider multiplicative noise as being able – like an outer force to produce resonance in the unperturbed system. We look after noise generated resonance versus non-resonance and noise induced rotation or combinations of both occuring, in particular, in systems of higher finite dimensions and infinite dimensions.

## Efficient minimum action method for capturing the most probable transition path

**Xiaoliang Wan**

**Louisiana State University**

**Abstract:** Minimum action method (MAM) plays an important role in minimizing the Freidlin-Wentzell action functional, which is the central object of the Freidlin-Wentzell theory of large deviations for small-noise-induced transitions in stochastic dynamical systems. Because of the demanding computation cost, especially in spatially extended systems, numerical efficiency is a critical issue for MAM. Difficulties come from both temporal and spatial discretizations. One severe hurdle for the application of MAM to large scale systems is the global reparametrization in time direction, which is needed in most versions of MAM to achieve accuracy. We have developed a new version of MAM based on optimal linear time scaling where the global reparametrization is replaced by hp adaptivity guided by a posteriori error estimates. More specifically, we use the zero-Hamiltonian constraint to define an indicator to measure the error induced by linear time scaling, and the derivative recovery technique to construct an error indicator and a regularity indicator for the transition paths approximated by finite elements.

**On a class of stochastic differential equations with local times adjustment**

**Yongjin Wang**

**Nankai University**

**Abstract：** In this talk, we shall address a class of stochastic differential equations with local times adjustment, which actually conduct a class of stochastic processes with partial reflections, also called skew diffusion processes.  We start at the skew Brownian motion, and describe its construction and the corresponding SDE with local time. Next we are going to study skew Ornstein-Unlenbeck processes and skew Feller branching processes. We are going to derive  their probability transition densities expressions and the Laplace transforms of  the first Hitting times. Those quantities are playing fundamental roles in the  analysis of the stochastic processes.

**Multi-valued non-autonomous random dynamical systems**

**Yejuan Wang**

**Lanzhou University**

**Abstract：** We first present a sufficient and necessary condition for the existence of pullback attractors of multi-valued non-compact random dynamical systems. We then prove the existence of pullback attractors for reaction-diffusion equations with non-autonomous deterministic as well as stochastic forcing terms for which the uniqueness of solutions need not hold. In particular, the existence of extremal complete trajectories is established for the multi-valued non-compact random dynamical system. Finally,the asymptotic behavior of stochastic systems with a Caputo fractional time derivative is investigated, and the existence of a global forward attracting set in the mean-square topology is given.

## Stochastic Schrodinger Equations Associated with Bernoulli Annihilation

## and Creation Operators

**Caishi Wang**

**Northwest Normal University**

(Joint work with Jinshu Chen)

**Abstract**： Bernoulli annihilation and creation operators are annihilation and creation operators acting on Bernoulli functionals, which satisfy a canonical anti-commutation relation (CAR) in equal-time. In this talk, we consider linear stochastic Schrodinger equations (LSSE) associated with Bernoulli annihilation and creation operators in the space of square integrable Bernoulli functionals. Among others, we obtain an easily-checking condition for such a equation to have a unique Nr-strong solution of mean square norm conservation for given , where is the number operator acting on Bernoulli functionals.

## A diffusion approximation for random nonlinear wave equation with large interaction

**Wei Wang**

**Nanjing University**

**Abstract**： This talk presents a diffusion approximation for random nonlinear wave equation with large interaction and small mass. The limit is shown to be a

one dimensional one order stochastic differential equation. The boundary with a randomly high oscillation is also considered.

## Data assimilation scheme for decadal predictions using a coupled climate system model

**Bin Wang**

**Tsinghua University & Chinese Academy of Sciences**

(Joint work with Yujun He, Mimi Liu)

**Abstract**： The initialization for decadal predictions using a coupled model includes a long-term cycle of data assimilation in the coupling framework, which is an essential step of decadal prediction. The skill of decadal prediction closely relies on the performance of the assimilation scheme used in the initialization. Most decadal prediction experiments conducted in the Coupled Model Intercomparsion Project Phase 5 (CMIP5) simply used the nudging approach, and very few applied the variational or ensemble methods that are very expensive in computing. Therefore, it is necessary to develop an assimilation scheme that can well use the observations in an economical way. In this paper, we propose a coupled assimilation scheme that can incorporate 4-dimensional ocean observations (say temperature and salinity) into the IC of ocean component model and in the meantime can be time-saving. Numerical experiments using a coupled climate system model are conducted to test the performance of the new assimilation scheme.

## The response analysis of fractional-order stochastic system via generalized cell mapping method

**Liang Wang**

**Northwestern Polytechnical University**

(Joint work with Lili Xue)

**Abstract**：We research the response of the fractional-order stochastic system via generalized cell mapping method. The short memory principle is introduced to ensure that the response of the system is a Markov process. The generalized cell mapping method is applied to display the global dynamics of the noise-free system. The stochastic generalized cell mapping method is employed to obtain the evolutionary process of probability density functions of the response. The fractional-order Duffing system and the SD oscillator with fractional derivative damping are taken as examples to give the implementations of our strategies discussed above. Studies have shown that the evolutionary direction of PDF of fractional-order stochastic system is consistent with the unstable manifold. We also found that the smoothness parameter, the noise intensity, the amplitude and frequency of the harmonic force can induce the occurrence of stochastic P-bifurcation in the SD oscillator. The effectiveness of the method is confirmed using Monte Carlo simulations.

## Random attractors for stochastic differential equations driven by two-sided Lévy processes

**Yong Xu**

**Northwestern Polytechnical University**

（Joint work with Xiaoyu Zhang and Bjoern Schmalfuss）

**Abstract**：At present, the study of random attractors mainly concentrate on the stochastic differential equations driven by Gaussian process. However, there are much complex phenomena perturbed by non-Gaussian noise especially Lévy noise in practice. Lévy process, as a typical type of non-Gaussian process with jumps, can describe the discontinuous complex phenomena with big jumps accurately. Up to now the research on the relevant basic theories of random attractors of the systems driven by Lévy process has far-reaching implications since the stochastic integral should be re-defined. In this talk, the random attractors and random bifurcations in random dynamical systems which are solutions of nonlinear Marcus type stochastic differential equations with multiplicative Lévy noise are investigated.

## Random Transformations and its applications to Synchronization

华中科技大学数学中心
Center for Mathematical Sciences

**Xiaofeng Ye**

**University of Washington**

**Abstract:** We establish some elementary contradistinctions between Markov chain (MC) and RDS descriptions of a stochastic dynamics. It is shown that a given MC is compatible with many possible RDS, and we study in particular the corresponding RDS with maximum metric entropy. Specifically, we show an emergent behavior of an MC with a unique absorbing and aperiodic communicating class, after all the trajectories of the RDS synchronizes.

We also consider a finite state hidden Markov model (HMM) with multidimensional observations. Under some mild assumptions, the prediction filter forget almost surely the initial condition exponentially fast. However, it is very difficult to calculate this asymptotic rate of exponential loss of memory analytically. We restate this problem as synchronization in random dynamical system and we use the Lyapunov exponents of the induced random dynamical system defined in the projective space $R^{n-1}$ to approximate the convergence rate. Finally, we propose a stable numerical algorithm to calculate the rate of exponential forgetting. The numerical simulation result and comparison with current upper bound in literature will be shown in the presentation.

**Periodic Stochastic Dynamical Systems (PeriSDS)**

**Huaizhong Zhao**

**Loughborough University**

(Joint work with C Feng)

**Abstract:** I will talk random periodic processes, periodic measures and their “equivalence”. I will also talk about their existence, ergodic theory and pure imaginary eigenvalues of the corresponding Markovian semigroup.

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